Wave functions and decay constants of B and D mesons in the relativistic potential model

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With the decay constants of D and D_s mesons measured in experiment recently, we revisit the study of the bound states of quark and antiquark in B and D mesons in the relativistic potential model. The relativistic bound state wave equation is solved numerically. The masses, decay constants and wave functions of B and D mesons are obtained. Both the masses and decay constants obtained here can be consistent with the experimental data. The wave functions can be used in the study of B and D meson decays.

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I Introduction

The wave function of the bound state of quark and antiquark is determined by the strong interaction between quark and antiquarks. The study of the wave functions of heavy-flavored mesons like B and D are important not only for studying the property of strong interaction between heavy and light quarks, but also for investigating the mechanism of heavy meson decays. The wave function determines the momentum distributions of the quark and antiquark in mesons, which is an important quantity for calculating the amplitude of heavy meson decays [1, 2]. The light-cone momentum distribution amplitudes for B meson appear in the amplitude of B decays, which are defined through the hadron-to-vacuum matrix element of non-local operators of quark and antiquark separated along the light-cone $\langle 0|\bar{q}^\beta(z)b^\alpha(0)|\bar{B}(p)\rangle$. The light-cone distribution amplitudes of B meson have been extensively studied in the recent several years. Some properties of the light-cone distribution amplitudes have been obtained. Based on these achievements, several models satisfying these constraints have been proposed in the literature [3–9]. Methods to obtain the light-cone distribution amplitude exactly from the first principle of QCD are still under investigation.

Alternatively, directly studying the wave functions of the heavy mesons by solving the bound state wave equation is an effective way to obtain the knowledge about the bound state of quark and antiquark [10–15]. For a heavy-light system, in the heavy quark limit, the heavy quark can be viewed as a static color source in the rest frame of the heavy meson. The light antiquark is bound around the heavy quark by an effective potential. The heavy quark spin decouples from the interactions as $m_Q \to \infty$ (m_Q is the mass of the heavy quark) [16, 17]. The interactions relevant to quark spin can be treated as perturbative correction.

Inspired by asymptotic freedom at short distance in QCD and quark confinement at long distance, the effective potential between quark and antiquark in meson can be taken as a combination of a Coulomb term and a linear confining term. Such a potential will be consistent with perturbative QCD at short distance, it can also generate quark confinement at long distance [11, 18].

The parameters in the effective potential can be constrained by comparing the eigenvalues of the bound state wave equation with the masses of the relevant bound states measured in experiment. Recently the decay constants of D and D_s mesons have been measured by CLEO [19–21], Belle [22] and *BABAR* [23] Collaborations. The measured values of D and D_s mesons' decay constants f_D and f_{D_s} can give further information about the interactions within the heavy-light quark-antiquark system. In this paper, with the recently measured decay constants f_D and f_{D_s} available, we revisit the study of the bound states of B, B_s, D and D_s mesons in the relativistic potential model. We solve the relativistic version of Schrödinger equation for the bound state wave function of heavy-light quark-antiquark meson system. The decay constants f_D and f_{D_s} can be used as a further constriant on the parameters in the potential model. The obtained masses and decay constants of B and D mesons can be well consistent with the values measured in experiments. Then the wave functions obtained here can be more reliable then ever. It can be useful for studying B and D decays, where the momentum-distribution of the quarks is needed.

Although the bound states of heavy mesons have been studied with the relativistic potential model in the literature several years before, these works need to be improved with the recent experimental data of the decay constants of D and D_s mesons available. The decay constants and bound state masses are calculated in Refs.[13–15], where Richardson potential [24] was taken, here the potential we considered is different from theirs. In addition, with the experimental values of the decay constants f_D and f_{D_s} available recently, the parameters in the potential can be constrained more stringently. Therefore our prediction on the decay constants for B and D mesons are quite different from previous predictions in the relativistic potential model.

The paper is organized as follows. In section II, we

solve the relativistic wave equation for the heavy-light quark-antiquark system. Section III gives the decay constant in terms of the wave function. In section IV the QCD-inspired potential is presented. Section V is devoted to the numerical result and discussion. Section VI is a brief summary.

II The relativistic wave equation for heavy-light system and the solvement

The B and D mesons are assumed to be approximately described in terms of heavy-light valence-quark configurations in the rest-frame of the mesons. The effective potential is one-gluon exchange dominant at short distances and a linear confinement at long distances [11]. The equation describing the bound state wave functions is a Schrödinger-type wave equation with relativistic dynamics [11–15]

$$\left[\sqrt{-\hbar^2\nabla_1^2+m_1^2}+\sqrt{-\hbar^2\nabla_2^2+m_2^2}+V(r)\right]\psi(\vec{r}) = E\psi(\vec{r}), \tag{1}$$

where $\vec{r}=\vec{x}_1-\vec{x}_2$ is the displacement of the light antiquark from the heavy quark, and $\vec{x}_1$ and $\vec{x}_2$ are the coordinates of the light and heavy quarks respectively. The operators ∇_1 and ∇_2 are the gradient operators relevant to the coordinates of $\vec{x}_1$ and $\vec{x}_2$. m_1 is the mass of the light antiquark, and m_2 the mass of the heavy quark. $V(r)$ is the effective potential of strong interaction between heavy and light quark-antiquark. In the rest frame of the bound state system, the eigenvalues in the wave equation will be the masses of the series bound states.

The wave function can be expressed in terms of spectrum integration

$$\begin{aligned}\psi(\vec{r}) &= \int d^3r'\delta^3(\vec{r}-\vec{r}\,')\psi(\vec{r}\,') \\ &= \int d^3r'\int\frac{d^3k}{(2\pi\hbar)^3}e^{i\vec{k}\cdot(\vec{r}-\vec{r}\,')/\hbar}\psi(\vec{r}\,').\end{aligned} \tag{2}$$

Substitute Eq.(2) into Eq.(1), the wave equation becomes

$$\int\frac{d^3k}{(2\pi\hbar)^3}d^3r'(\sqrt{k^2+m_1^2}+\sqrt{k^2+m_2^2}\,) \times e^{i\vec{k}\cdot(\vec{r}-\vec{r}\,')/\hbar}\psi(\vec{r}\,')=(E-V(r))\psi(\vec{r}). \tag{3}$$

The exponential $e^{i\vec{k}\cdot\vec{r}/\hbar}$ can be decomposed in spherical harmonics

$$e^{i\vec{k}\cdot\vec{r}/\hbar}=4\pi\sum_{ln}i^l j_l(\frac{kr}{\hbar})Y^*_{ln}(\hat{k})Y_{ln}(\hat{r}), \tag{4}$$

where $j_l(\frac{kr}{\hbar})$ is the spherical Bessel function, $Y_{ln}(\hat{r})$ is the spherical harmonics, which satisfies the normalization condition

$$\int d\Omega Y_{l_1n_1}(\hat{r})Y_{l_2n_2}(\hat{r})=\delta_{l_1l_2}\delta_{n_1n_2}. \tag{5}$$

Using the spherical harmonics decomposition of the exponential in Eq.(4), and factorize the wave function into the product of two parts: radial and angular wave functions

$$\psi(\vec{r})=\Phi_l(r)Y_{ln}(\hat{r}), \tag{6}$$

then the wave equation of Eq.(3) can be transferred to be

$$V(r)\Phi_l(r)+\frac{2}{\pi\hbar}\int dk\frac{k^2}{\hbar^2}\int dr'r'^2(\sqrt{k^2+m_1^2} +\sqrt{k^2+m_2^2}\,)j_l(\frac{kr}{\hbar})j_l(\frac{kr'}{\hbar})\Phi_l(r')=E\Phi_l(r). \tag{7}$$

For convenience later, let us define a new reduced radial wave function $u_l(r)$ by

$$\Phi_l(r)=\frac{u_l(r)}{r}. \tag{8}$$

With this definition, and for the case $l=0$ which we are interested in this work, Eq.(7) becomes

$$V(r)u_0(r)+\frac{2}{\pi\hbar}\int_0^\infty dk\int_0^\infty dr'(\sqrt{k^2+m_1^2} +\sqrt{k^2+m_2^2}\,)\sin(\frac{kr}{\hbar})\sin(\frac{kr'}{\hbar})u_0(r')=Eu_0(r), \tag{9}$$

where we have used the explicit expression of the spherical Bessel function for $l=0$

$$j_0(x)=\frac{\sin x}{x}. \tag{10}$$

Eq.(9) is for taking $c=1$, if recover the speed of light appearing in the formulas, Eq.(9) should be

$$V(r)u_0(r)+\frac{2}{\pi\hbar c}\int_0^\infty dk\int_0^\infty dr'(\sqrt{k^2+m_1^2} +\sqrt{k^2+m_2^2}\,)\sin(\frac{kr}{\hbar c})\sin(\frac{kr'}{\hbar c})u_0(r')=Eu_0(r). \tag{11}$$

In principle the integration over momentum k in the above equation can be performed because the wave function $u_0(r')$ does not depend on the momentum. However the integration over k will give a singular term for $r'\to r$ in the above equation [14]. In this work, we will take a new step to continue to solve this equation, this method can circumvent the appearance of the singular integral equation.

For a bound state of two particles, when the separation between them is large enough, the wave function will effectively vanish. We assume such a large enough typical value for the separation between the heavy quark and

the light antiquark is L, then the quark-antiquark in the bound state can be approximately treated as if they are restricted in a limited space $0 < r < L$. In the limited space, the Fourier expansion of the reduced wave function $u_0(r)$ is

$$u_0(r) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi}{L} r\right), \tag{12}$$

where the expansion coefficients c_n are

$$c_n = \frac{2}{L} \int_0^L \sin\left(\frac{n\pi}{L} r\right) u_0(r) dr. \tag{13}$$

In the limited space, the momentum k should be discretized, the integration over k should be replaced by a summation, the following substitution should be made in the wave equation (11)

$$\frac{k}{\hbar c} \to \frac{n\pi}{L}, \qquad \int \frac{dk}{\hbar c} \to \frac{\pi}{L}. \tag{14}$$

With the above replacement, and the integration over the distance r' being limited within $0 < r' < L$, Eq.(11) becomes

$$\begin{aligned} & V(r)u_0(r) + \sum_n \frac{2}{L} \int_0^L dr' \left(\sqrt{\left(\frac{n\pi\hbar c}{L}\right)^2 + m_1^2} \right. \\ & \left. + \sqrt{\left(\frac{n\pi\hbar c}{L}\right)^2 + m_2^2} \right) \sin(\frac{n\pi}{L} r) \sin(\frac{n\pi}{L} r') u_0(r') \\ & = E u_0(r). \end{aligned} \tag{15}$$

The above equation can go back to Eq.(11) as $L \to \infty$. Numerically if the value of L is taken to be large enough, the solution of this equation only slightly depends on the value of L. For the parameters we take in section V, we find that the solution of the wave equation will be stationary when $L > 5$ fm.

Truncate the series of the Fourier expansion of the wave function $u_0(r)$ as

$$u_0(r) = \sum_{n=1}^{N} c_n \sin\left(\frac{n\pi}{L} r\right), \tag{16}$$

where N is a large integer. Substitute this truncated expansion into the wave equation (15) and simplify it, one can finally get the equation about c_n

$$\begin{aligned} & \left(\sqrt{\left(\frac{n\pi\hbar c}{L}\right)^2 + m_1^2} + \sqrt{\left(\frac{n\pi\hbar c}{L}\right)^2 + m_2^2} \right) c_n \\ & + \sum_{m=1}^{N} \frac{2}{L} \int_0^L dr V(r) \sin\left(\frac{n\pi}{L} r\right) \sin\left(\frac{m\pi}{L} r\right) c_m \\ & = E c_n. \end{aligned} \tag{17}$$

The above equation is just the eigenstate equation in the matrix form. It is not difficult to solve it numerically. The eigenvalues are the masses of the series of bound states of the heavy-light quark-antiquark system. Once the eigen equation is solved, the eigenvectors composed of c_n can be substituted into Eq.(16) to get the reduced wave function $u_0(r)$.

To get the wave function in momentum space, one can use the Fourier transform of the wave function $\psi(\vec{r})$

$$\Psi(\vec{k}) = \frac{1}{(2\pi\hbar c)^{3/2}} \int d^3 r e^{-i\vec{k}\cdot\vec{r}/\hbar c} \psi(\vec{r}). \tag{18}$$

Separate the variable-dependence of the momentum-space wave function as

$$\Psi(\vec{k}) = \Psi_l(k) Y_{lm}(\theta, \phi). \tag{19}$$

As in Eq.(8), we define the reduced wave function in momentum space

$$\Psi_l(k) = \frac{\varphi_l(k)}{k}. \tag{20}$$

Then using Eqs.(4), (6), (8), (19) and (20), one can derive from Eq.(18)

$$\varphi_l(k) = (-i)^l \sqrt{\frac{2}{\pi\hbar c}} \int_0^\infty dr \frac{kr}{\hbar c}\, j_l(\frac{kr}{\hbar c}) u_l(r). \tag{21}$$

For the case $l = 0$, we get

$$\varphi_0(k) = \sqrt{\frac{2}{\pi\hbar c}} \int_0^\infty dr \sin(\frac{kr}{\hbar c}) u_0(r), \tag{22}$$

which gives the momentum distribution of the quark and antiquark in the rest frame of the heavy meson.

III The pseudoscalar bound state of heavy-light system and the decay constant

The pseudoscalar meson composed of a heavy quark and a light antiquark $Q\bar{q}$ (Q can be b or c quark, q stands for u, d or s quark) can be written in the meson rest frame as follows

$$\begin{aligned} |P(\vec{p}=0)\rangle = & \frac{1}{\sqrt{3}} \sum_i \int d^3 k \Psi_0(k) \frac{1}{\sqrt{2}} [b_Q^{i+}(\vec{k},\uparrow) d_q^{i+}(-\vec{k},\downarrow) \\ & - b_Q^{i+}(\vec{k},\downarrow) d_q^{i+}(-\vec{k},\uparrow)] |0\rangle, \end{aligned} \tag{23}$$

where i is the color index. The factor $1/\sqrt{3}$ is the normalization factor for color indices, and $1/\sqrt{2}$ the normalization factor for spin indices.

The normalization of the meson state is

$$\langle P(\vec{p}_1) | P(\vec{p}_2) \rangle = (2\pi)^3 2E \delta^3(\vec{p}_1 - \vec{p}_2), \tag{24}$$

where E is the energy of the meson.

Substituting Eq.(23) into Eq.(24), we can finally get the normalization condition of the wave function in momentum space

$$\int d^3k|\Psi_0(k)|^2 = (2\pi)^3 2E. \tag{25}$$

The decay constant of a pseudoscalar is defined by the hadron-to-vacuum matrix element of the axial current

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 Q|P(p)\rangle = if_P\, p_\mu. \tag{26}$$

Substituting the meson state of Eq.(23) into the above equation in the rest frame, and contracting the quark (antiquark) creation operators in the meson state with quark (antiquark) annihilation operators in the quark field of the axial current, we can get the expression of the pseudoscalar decay constant

$$\begin{aligned} f_P \;=\; & \sqrt{\frac{3}{2}}\frac{1}{2\pi^2}\frac{1}{m_P}\int_0^\infty dk|\vec{k}|^2\Psi_0(k) \\ & \times\frac{(E_q+m_q)(E_Q+m_Q)-|\vec{k}|^2}{\sqrt{E_qE_Q(E_q+m_q)(E_Q+m_Q)}}, \end{aligned} \tag{27}$$

where E_Q and E_q are the energy of the heavy and light quarks. To be consistent with the wave equation, here both heavy and light quarks are taken to be on-shell. We assume that the decays of the heavy meson can be approximately described in terms of on-shell valence quarks, although the sum of the four-momenta of the valence quarks are not equal to that of the meson because of the existence of the color field within the hadron which can carry both energy and momentum.

We would like to mention that the leptonic decay of pseudoscalar mesons was considered several decades ago in a different method by assuming the coupling of meson with quark-antiquark pair [25].

IV The QCD-inspired potential

The potential of strong interaction between the heavy quark and light antiquark is taken as a combination of a Coulomb term and a linear confining term inspired by QCD [11, 18]

$$V(r) = -\frac{4}{3}\frac{\alpha_s(r)}{r} + b\,r + c. \tag{28}$$

The first term is the Coulomb term, which is consistent with one-gluon-exchange contribution for short distance calculated in perturbative QCD. The second term is the linear-confinement term, which generates confinement in long distance. The third term is a phenomenological constant, which is needed to reproduce the correct masses for heavy-light meson system.

The running coupling constant $\alpha_s(Q^2)$ in momentum space with N_f quark flavors at large values of Q^2, calculated in lowest-order QCD, is

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2N_f)\ln(Q^2/\Lambda^2)}. \tag{29}$$

This behavior of the strong coupling can be parameterized in a simpler form which can be conveniently transformed into the r-space [11]

$$\alpha_s(Q^2) = \sum_i \alpha_i e^{-Q^2/4\gamma_i^2}, \tag{30}$$

where α_i are free parameters chosen to fit the behavior of $\alpha_s(Q^2)$ given by perturbative QCD (Eq.(29)). As $Q^2 \to \Lambda^2$, the coupling $\alpha_s(Q^2)$ diverges, which is believed to be a signal of confinement. However, as $Q^2 \to \Lambda^2$, perturbative QCD can not apply, the behavior of $\alpha_s(Q^2)$ at small Q^2 given in Eq.(29) cannot be the exact prediction of QCD. One can make other choice for the behavior of the strong coupling at small momentum transfer. As in Ref. [11], we assume that the coupling α_s saturate as a critical value $\alpha_s^{\rm critical}$, where $\alpha_s^{\rm critical} = \sum_i \alpha_i$. In practice, only several α_i are needed to be non-zero, which can fit the behavior of $\alpha_s(Q^2)$ well at perturbative region, deviation only occurs at small Q^2.

The transformation of $\alpha_s(Q^2)$ by using Eq. (30) instead of Eq. (29) is [11]

$$\alpha_s(r) = \sum_i \alpha_i \frac{2}{\sqrt{\pi}}\int_0^{\gamma_i r} e^{-x^2}dx. \tag{31}$$

Fig.1 is the behavior of $\alpha_s(r)$ with the parameters $\alpha_1 = 0.15$, $\alpha_2 = 0.15$, $\alpha_3 = 0.20$, and $\gamma_1 = 1/2$, $\gamma_2 = \sqrt{10}/2$, $\gamma_2 = \sqrt{1000}/2$, which is relevant to the critical value $\alpha_s^{\rm critical} = 0.5$.

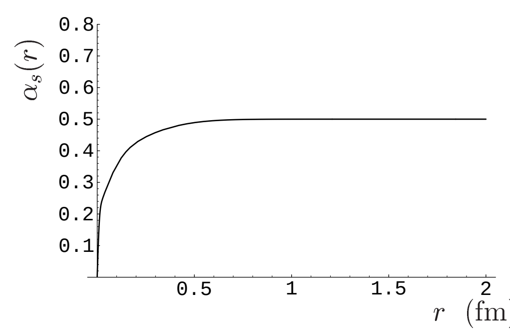

FIG. 1: The behavior of $\alpha_s(r)$, with the parameters $\alpha_1 = 0.15$, $\alpha_2 = 0.15$, $\alpha_3 = 0.20$, and $\gamma_1 = 1/2$, $\gamma_2 = \sqrt{10}/2$, $\gamma_2 = \sqrt{1000}/2$.

V Numerical result and discussion

The parameters are selected by comparing the predicted heavy meson mass with the experimental data.

The recently measured values of the decay constants of D and D_s mesons can give a further constraint on the parameters. The parameters which we finally obtain are

$$
\begin{aligned}
& b = 0.10\ \mathrm{GeV}^2, \quad c = -0.19\ \mathrm{GeV}^2, \\
& m_b = 4.98\ \mathrm{GeV}, \quad m_c = 1.54\ \mathrm{GeV}, \\
& m_s = 0.30\ \mathrm{GeV}, \quad m_u = m_d = 0.08\ \mathrm{GeV}, \\
& \alpha_s^{\mathrm{critical}} = 0.5,
\end{aligned} \tag{32}
$$

and $L = 10$ fm, $N = 100$.

The masses and decay constants of the B, B_s, D and D_s mesons calculated with the above parameters will be given in the following. The masses are given in Table I. Here we do not consider the contribution of spin-dependent interactions in our calculation, it may give errors about $100 \sim 200$MeV for the masses. Varying the parameters may also give errors to the numerical values. We estimate the combination of both the errors can be about 7% for B and B_s mesons, and 10% for D and D_s mesons.

TABLE I: Masses of pseudoscalar heavy mesons calculated by solving the wave equation, and the comparison with experimental data. The data is quoted from the Particle Data Group [26].

	m_B	m_{B_s}	m_D	m_{D_s}
this work (GeV)	5.25 ± 0.37	5.34 ± 0.37	1.86 ± 0.19	1.96 ± 0.20
Exp. (MeV)	5279.17 ± 0.29	5366.3 ± 0.6	1869.6 ± 0.16	1968.47 ± 0.33

The decay constants obtained are

$$
\begin{aligned}
& f_B = 198 \pm 14\ \mathrm{MeV}, \quad f_{B_s} = 237 \pm 17\ \mathrm{MeV}, \\
& f_D = 208 \pm 21\ \mathrm{MeV}, \quad f_{D_s} = 256 \pm 26\ \mathrm{MeV}.
\end{aligned} \tag{33}
$$

The comparison of the decay constants obtained in this work with experimental data are given in Table II. Both the masses and decay constants obtained in this work can be well consistent with experiment. For f_B and f_{B_s}, there are still no precise measured values in experiments yet. Our prediction can be tested in experiment in the future.

The leptonic decay rates of B meson relevant to the decay constant f_B obtained in this work are

$$Br(B^+ \to e^+\nu_e) = (1.11 \pm 0.26) \times 10^{-11}, \tag{34}$$
$$Br(B^+ \to \mu^+\nu_\mu) = (4.7 \pm 1.1) \times 10^{-7}, \tag{35}$$
$$Br(B^+ \to \tau^+\nu_\tau) = (1.1 \pm 0.2) \times 10^{-4}, \tag{36}$$

where the errors are mainly caused by the uncertainties of the decay constant f_B and the CKM matrix element V_{ub}. The value of $|V_{ub}|$ is quoted from PDG [26]

$$|V_{ub}| = (3.93 \pm 0.36) \times 10^{-3}.$$

At present the branching ratio of $Br(B^+ \to \tau^+\nu_\tau)$ has been measured in experiment. The results still suffer from large uncertainties. The measured value of Belle collaboration is $Br(B^+ \to \tau^+\nu_\tau) = (1.79^{+0.56+0.46}_{-0.49-0.51}) \times 10^{-4}$ [28], while the values of $BABAR$ collaboration are $Br(B^+ \to \tau^+\nu_\tau) = (0.9 \pm 0.6 \pm 0.1) \times 10^{-4}$ [29] and $(1.8^{+0.9}_{-0.8} \pm 0.4 \pm 0.2) \times 10^{-4}$ [30]. The combined result of $BABAR$ collaboration is $Br(B^+ \to \tau^+\nu_\tau) = (1.2 \pm 0.4 \pm 0.3 \pm 0.2) \times 10^{-4}$ [31]. Considering the large uncertainties of the experimental results, our predictied branching ratio of the decay mode $B^+ \to \tau^+\nu_\tau$ is cosistent with the experimental data.

A super B factory will come into operation with the designed peak luminosity in excess of 10^{36} cm^{-2}S^{-1} at the $\Upsilon(4s)$ resonance in the next half decades [32]. The integrated luminosity of 75 ab^{-1} would be collected in five years of data taking. Then the branching ratios of $B^+ \to \tau^+\nu_\tau$ and $\mu^+\nu_\mu$ can be measured at the Super$B$ factory with precisions of up to 4% and 5% respectively [32]. Taking the value of the CKM matrix element $|V_{ub}|$ as input, the decay constant of f_B can be obtained at super$B$.

The decay constants are also compared with previous theoretical results in Table III. Our predictions for the decay constants are quite different from previous results calculated in the relativistic potential model. For f_D and f_{D_s}, our results are larger than that in Ref [14], while our results for f_B and f_{B_s} are smaller than theirs. [45]

TABLE II: Decay constants of pseudoscalar heavy mesons calculated in this work, and the comparison with experimental data. All values are in units of MeV.

	f_B	f_{B_s}	f_D	f_{D_s}
this work	198 ± 14	237 ± 17	208 ± 21	256 ± 26
Exp. [19, 27]	–	–	$205.8 \pm 8.5 \pm 2.5$	254.6 ± 5.9

The reduced wave functions in coordinate and momentum spaces (Eqs. (8) and (20) are depicted in Figs. 2 and 3. The wave function squared $|u_l(r)|^2$ is the possibility density distributed along the quark-antiquark distance r. The curves in Fig. 2 show that the most probable distribution of the quarks occurs at the distance 0.4 fm between the quark and antiquark in both B and D mesons. The possibility density vanishes as the distance larger than 2 fm. The mean square root of the distance is about $0.5 \sim 0.7$ fm.

The numerical solution of the wave function in momentum space is given in Fig.3, which shows that the peak of the momentum-distribution of the quarks in the heavy meson is at about 0.4 GeV. The reduced wave function can be fitted with the analytical form as suggested in Ref.[15]

$$\varphi_0(k) = 4\pi\sqrt{m_H \alpha^3} k e^{-\alpha k}, \tag{37}$$

TABLE III: The comparison of the decay constants calculated in this work with other theoretical results. All values are in units of MeV.

	f_B	f_{B_s}	f_D	f_{D_s}
this work	198 ± 14	237 ± 17	208 ± 21	256 ± 26
Ref.[14][a]	230 ± 35	245 ± 37	182 ± 27	199 ± 30
Ref.[33][b]	203 ± 23	236 ± 30	205 ± 20	235 ± 24
Ref.[34][b]	206 ± 20	–	195 ± 20	–
Ref.[35][c]	–	–	200 230	221 270
Ref.[36][d]	–	–	177 ± 21	205 ± 22
Ref.[37][e]	193	195	238	241
Ref.[38][e]	196 ± 29	216 ± 32	230 ± 25	248 ± 27
Ref.[39][f]	189	218	234	268
Ref.[40][g]	210 ± 11.4 ± 5.7	–	–	–
Ref.[41][h]	–	–	201 ± 3 ± 17	249 ± 3 ± 16
Ref.[42][h]	195 ± 11	243 ± 11	207 ± 11	249 ± 11
Ref.[43][h]	–	–	207 ± 4	241 ± 3
Ref.[44][h]	–	–	–	248.0 ± 2.5

a Relativistic potential model.
b QCD sum rule.
c Light front quark model.
d Finite energy sum rules.
e Quark model based on Bethe-Salpeter equation.
f Relativistic constituent Quark Model.
g Derived from result of Lattice QCD.
h Lattice QCD

where m_H is the heavy meson mass. The factor $4\pi\sqrt{m_H\alpha^3}$ is the normalization factor due to the normalization condition in Eq.(25). Note that the wave function for B and/or D meson is $\Psi_0(k) = \varphi_0(k)/k$. Our numerical solution gives $\alpha = 3.0\ \mathrm{GeV}^{-1}$, $2.6\ \mathrm{GeV}^{-1}$, $3.4\ \mathrm{GeV}^{-1}$, and $3.2\ \mathrm{GeV}^{-1}$ for B, B_s, D and D_s mesons, respectively.

With the constraint of the measured values of f_D and f_{D_s} considered, the wave functions obtained here can be more reliable than before, which should be useful in studying the decays of the B and D mesons. The application of the wave functions in studying the heavy meson decays deserves a separate work.

VI Summary

The wave functions and decay constants of B and D mesons are revisited in the relativistic potential model. The parameters in the potential model are further constrained with the experimental values of f_D and f_{D_s} available. The masses and decay constants of the heavy mesons are obtained, which can be well consistent with the current experimental data. The wave functions both in coordinate and momentum spaces are obtained. The wave functions obtained here can be useful for studying heavy meson decays.

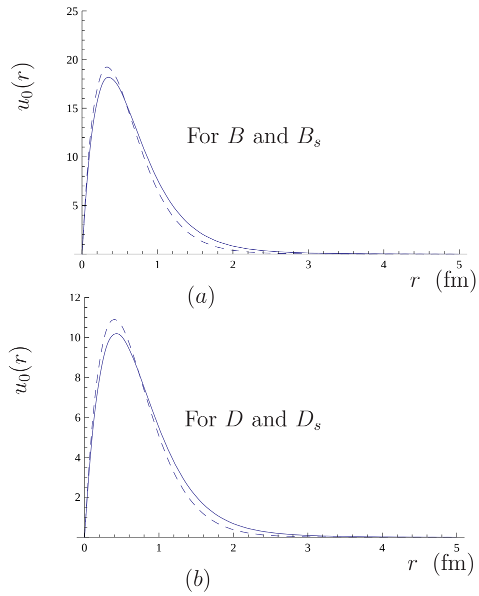

FIG. 2: The reduced wave function $u_0(r)$ in coordinate space. (a) is for B and B_s mesons. The solid curve is for the wave function of B meson, the dashed one is for B_s. (b) is for D and D_s mesons. The solid curve is for D, and the dashed one for D_s.

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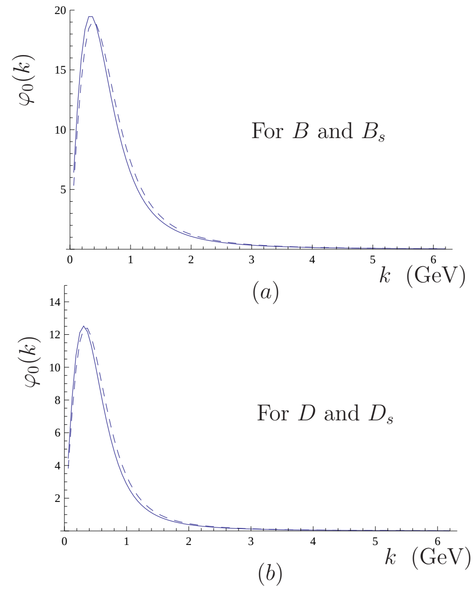

FIG. 3: The reduced wave function $\varphi_0(k)$ in momentum space. (a) is for B and B_s mesons. The solid curve is for B meson, the dashed one is for B_s. (b) is for D and D_s mesons. The solid curve is for D, and the dashed one for D_s.

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